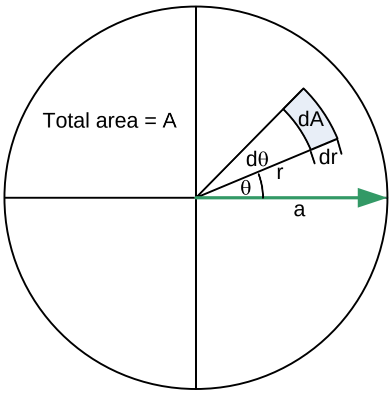
Minimum Hauling Distance Calculation

Case 1: Round field, straight transport



Let's consider a small patch of field dA.

$$dA = r.dr.d\theta$$

The crop harvested from *dA* will have to travel a distance of r to get to the center. Therefore total hauling distance traveled by harvesting the entire field:

$$Total\ hauling\ distance = \iint rdA$$

$$Total\ hauling\ distance = \int_0^{2\pi}\int_0^a r^2dr\,d\theta = \frac{2}{3}\pi a^3$$

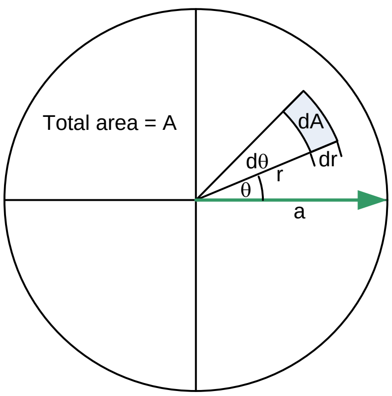
If the average hauling distance is *d* then, $d = \frac{Total\ hauling\ distance}{Total\ area}$

But, $Total\ area = \pi a^2$. Therefore, $d = \frac{\frac{2}{3}\pi a^3}{\pi a^2} = \frac{2}{3}a$

For a area of A, $a = \sqrt{\frac{A}{\pi}}$, therefore, $d = \frac{2}{3\sqrt{\pi}}\sqrt{A} = 0.376\sqrt{A}$

Make note that **the average hauling distance increases in proportion to the square root of area being covered**.

Case 2: Round field, NS-EW transport



The crop harvested from *dA* will have to travel a distance of $|rcos\theta| + |rsin\theta|$ to get to the center. We take absolute values because as θ is > 90°, the cosine terms goes to negative. Since travel distance cannot be negative, an absolute value was taken.

$$Total\ hauling\ distance = \iint rdA$$

$$Total\ hauling\ distance = \int_0^{2\pi}\int_0^a (|rcos\theta| + |rsin\theta|)rdr\,d\theta$$

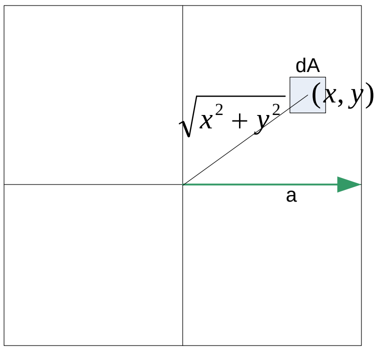
From symmetry property, we can write the above equation as:

$$Total\ hauling\ distance = 4\int_0^{\frac{\pi}{2}}\int_0^a (rcos\theta + rsin\theta)rdr\,d\theta = \frac{8}{3}a^3$$

hence avoiding the integration over absolute values. Therefore

$$d = \frac{\frac{8}{3}a^3}{\pi a^2} = \frac{8}{3\pi}a = \frac{8}{3\pi\sqrt{\pi}}\sqrt{A} = 0.479\sqrt{A}$$

Case 3: Square field, straight transport



The crop harvested from *dA* will have to travel a distance of $\sqrt{x^2 + y^2}$ to get to the center.

$$Total\ hauling\ distance = \int_{-a}^{a}\int_{-a}^{a}\sqrt{x^2+y^2}dx\,dy$$

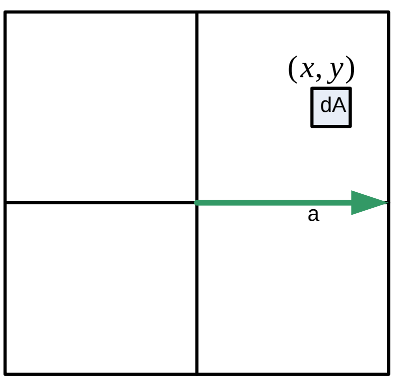
It is difficult to evaluate this integration as it is. Therefore we will assign the value of A = 1 and integrate it numerically. Since we know that the hauling distance is proportional to the square root of A (See the discussion for case 1), the hauling distance for area A will be the hauling distance for unit area multiplied by the square root of A.

For A = 1, a = 0.5

$Total\ hauling\ distance = \int_{-0.5}^{0.5}\int_{-0.5}^{0.5}\sqrt{x^2+y^2}dx\,dy$= 0.3826. Since A =1, Total hauling distance is same as average hauling distance.

The average hauling distance for any area A = $0.3826\sqrt{A}$

Case 4: Square field, NS-EW transport



The crop harvested from *dA* will have to travel a distance of $|x| + |y|$ to get to the center.

$$Total\ hauling\ distance = \int_{-a}^{a}\int_{-a}^{a}(|x|+|y|)dx\,dy$$

Since four quadrants are symmetrical

$$Total\ hauling\ distance = 4\int_{0}^{a}\int_{0}^{a}(x+y)dx\,dy$$

For a = 0.5,

$$Total\ hauling\ distance = 4\int_{0}^{0.5}\int_{0}^{0.5}(x+y)dx\,dy = 0.5$$

Average hauling distance for any area A = 0.5 $\sqrt{A}$